Averaged time evolution of rough surfaces

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We propose a method of extracting an approximated evolution equation of distribution of height fluctuations for a class of discrete growth models of rough surfaces where the behavior of local slopes can be described by a Markov process. Using the Markov property of the fluctuating field, the evolution equation is obtained as a map describing the time evolution averaged over possible growth paths. Applying the method to a $(1+1)$-dimensional restricted solid-on-solid model, we obtain a map with finite degrees of freedom. The obtained map describes well the averaged evolution of surfaces.

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I. INTRODUCTION

In the past couple of decades a lot of attention has been given to the formation of rough surfaces under nonequilibrium conditions [1]. One of intriguing features of the surface roughening is a self-organized nature of the growth. An initially flat surface evolves to a self-affine one characterized by the roughness exponent α in the Family-Vicsek scaling ansatz [2] without external parameter tuning. One way to study the pattern selection mechanism and its self-organized nature is to investigate the evolution of probability distribution for height difference between two positions, $\Delta h = h(x_1) - h(x_2)$. Recently, Jafari *et al.* investigated a Markov property of fluctuating height differences of a deposited copper film [3]. For the probability density function $P(\Delta h, \Delta x)$ in terms of the length scale Δx, they have checked the validity of the Chapman-Kolmogorov equation

$$p(\Delta h_2,\Delta x_2|\Delta h_1,\Delta x_1) = \int d(\Delta h_3) p(\Delta h_2,\Delta x_2|\Delta h_3,\Delta x_3) \times p(\Delta h_3,\Delta x_3|\Delta h_1,\Delta x_1), \qquad (1)$$

where $p(\Delta h_i, \Delta x_i|\Delta h_j, \Delta x_j)$ is a conditional probability. Satisfying Eq. (1), for any value of Δx_3 in the interval $\Delta x_2 < \Delta x_3 < \Delta x_1$, is a necessary condition for the field to be a Markov process in spatial dimensions [4]. Using the method proposed by Friedrich and Peinke [5,6], they have obtained a Fokker-Plank equation by measuring the Kramers-Moyal coefficients and shown that the obtained equation regenerates surfaces with similar statistical properties. The stochastic analysis based on the theory of the Markov process provides us another point of view to study the interfacial problem [7,8].

In this work, our attention is focused on the growth process generating rough surfaces showing the Markovian behavior and we propose a method of extracting an approximated evolution equation of distribution of height differences in such a system. In a previous paper [9], we have introduced the probable growth (PG) in discrete growth models to investigate the behaviors of relaxation to the steady state for arbitrary initial conditions, where the PG means the time evolution averaged over all possible growth paths starting with a given surface. The PG path is uniquely determined for a given surface. This enables us to introduce a deterministic equation into the stochastic growth. We have also clarified that the number of subsets of height differences, $n_t(\{\Delta h\})$, contained in the surface at time t, plays a role of a dynamical variable for the evolution equation associated with the PG in a certain class of growth models. In this paper we extend this idea to a restricted solid-on-solid (RSOS) model [10], which is a prototype generating rough surfaces without overhangs. For sufficiently large system size L, $n_t(\{\Delta h\})/L^d$ gives an estimation for the probability of finding $\{\Delta h\}$ in the whole surface in $d+1$ dimensions. Thanks to the Markov property, the probability of finding a given surface can be obtained from a product of conditional probabilities. Then the problem of identifying the probability distribution of height differences can be reduced to obtaining the conditional probabilities or $n_t(\{\Delta h\})$. Considering the PG, the evolution equation of $n_t(\{\Delta h\})$ can be simplified as a map with finite degrees of freedom. A fixed point of the map gives an estimator for the distribution of height differences in the steady state.

II. A MAP ASSOCIATED WITH THE PROBABLE GROWTH

In this section, we present a map with finite degrees of freedom to describe the PG of $(1+1)$-dimensional RSOS interfaces for arbitrary initial conditions. It is worthy to reconsider the interfacial problem in $1+1$ dimensions to illustrate our method although the problem has been extensively studied previously.

The growth algorithm of the RSOS model is as follows: Select randomly a site on a one-dimensional lattice and permit growth by letting the height h_i of the interface at site i increase by one unit, provided that the RSOS restriction on the height differences $|\Delta h|=0,1,\ldots,N$ is obeyed at all stages. We set hereafter $N=1$ for simplicity and our results can be straightforwardly extended to the case of $N>1$. One can interpret the algorithm for the RSOS growth as that for the dynamics of height differences. Denoting the height difference by s, the growth of $h_i \mapsto h_i+1$ corresponds to the change of a pair of successive variables, $(s_i,s_{i+1}) \mapsto (s_i+1,s_{i+1}-1)$. We here use periodic boundary conditions. The growth algorithm of the system is to randomly select a pair (s_i,s_{i+1}) and to perform the change provided that $|s_i|=0$ or 1 for $i=1,2,\ldots,L$ is satisfied at all stages. An allowable change for (s_i,s_{i+1}) is limited in the set $D=\{(-1,0),(0,1),(0,0),(-1,1)\}$ by the RSOS restriction.

Under the mapping between height differences and particles on a one-dimensional lattice, this algorithm corresponds to an extended process of the two-species asymmetric exclusion model (ASEP) on a ring, which contains right-moving particles $s=-1$ and left-moving particles $s=1$ with pair annihilation of particles, $(1,-1)\mapsto(0,0)$, and pair creation of particles, $(0,0)\mapsto(1,-1)$. The two-species ASEP, without creation and annihilation of particles, can be solved exactly by using a matrix formulation [11,12]. The steady state of the ASEP depends on an initial density of particles in a ring geometry since the number of particles are conserved during the time evolution. In contrast, the density of particles in the steady state of the RSOS model is determined in a self-organized way through creation and annihilation of particles. The behaviors of relaxation to the steady state differ from each other although both models belong to the same universality class.

We now consider the evolution of a set $\{s_1,s_2,\ldots,s_L\}$ in the $(1+1)$-dimensional RSOS model. Let $p_t(s_1,s_2,\ldots,s_L)$ be the probability of finding $\{s_1,s_2,\ldots,s_L\}$ at time t and $p_t(\{s_i\}|\{s_j\})$ be the conditional probability of finding a subsequence $\{s_i\}$ when $\{s_j\}$ is given, where t is equal to the mass of the cluster except a constant. We introduce the following relations into the conditional probabilities

$$p_t(s_{i+m}|\cdots,s_i,\overbrace{s_{i+1},\ldots,s_{i+m-1}}^{m-1}) = p_t(s_{i+m}|s_{i+1},\ldots,s_{i+m-1}), \tag{2a}$$

$$p_t(s_i|\overbrace{s_{i+1},\ldots,s_{i+m-1}}^{m-1},s_{i+m},\ldots) = p_t(s_i|s_{i+1},s_{i+2},\ldots,s_{i+m-1}). \tag{2b}$$

The interaction between separated sites is called the short-range interaction if m is finite, throughout this literature. The system can be reduced to a Markov process in length scales greater than m and the interaction length m corresponds to the Markovian length in Ref. [3].

We have numerically checked the Markov property of the process by calculating the correlation function

$$C_s(j) = \frac{\ll s_i s_{i+j} \gg}{\ll s_i^2 \gg}, \tag{3}$$

where the double angular brackets represent an ensemble average in the steady state. Figure 1 shows $C_s(j)$ obtained by averaging over 10^7 samples for $L=1000$. We have used free boundary conditions in calculations of $C_s(j)$. For periodic boundary conditions, the constraint $s_1+s_2+\cdots+s_L=0$ leads to $\Sigma_j C_s(j)=0$ and this relation affects on estimation of asymptotic behaviors since $C_s(j)$ becomes negative for large j. In Fig. 1 one can see that a crossover occurs at $j_c \sim 10$ and $C_s(j)$ decays exponentially for $j>j_c$. This implies that the interaction is of the short-range type. For systems in which the correlation length of height differences is finite, Eqs. (2a) and (2b) obviously hold for m sufficiently greater than the correlation length because two sites separated by m are independent of each other in a statistical sense. We comment on estimating the interaction length m in the case of algebraically decaying $C_s(j)$, which is expected in globally coupled interactions or $(2+1)$-dimensional growth. In this case the interaction length should be directly estimated by checking the validity of the Chapman-Kolmogorov equation for the conditional probabilities, or by measuring the Kramers-Moyal coefficients.

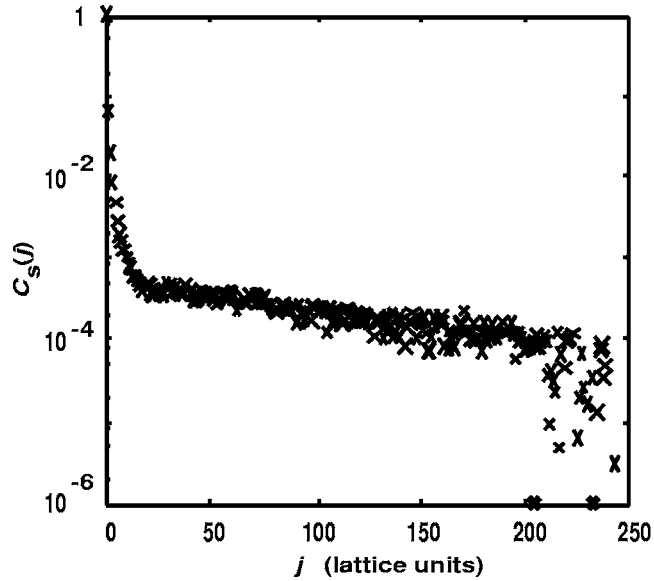

FIG. 1. Semilog plot of the correlation function $C_s(j)$ for $L=1000$.

Based on Eqs. (2a) and (2b), we can find the probability $p_t(s_1,\ldots,s_L)$ to be proportional to the product of conditional probabilities $\Pi_i p_t(s_{i+m-1}|s_i s_{i+1},\ldots,s_{i+m-2})$. Let C_m be a subsequence with length m and $n_t(C_m)$ be the number of C_m contained in $\{s_1,s_2,\ldots,s_L\}$ at time t. For $C_m=\{\sigma_1,\ldots,\sigma_m\}$, $n_t(C_m)$ is defined by

$$n_t(C_m) = \sum_{j=0}^{L-1}\left(\prod_{i=1}^{m}\delta_{s_{i+j},\sigma_i}\right), \tag{4}$$

where δ denotes the Kronecker's delta. Note that $n_t(C_k)$ for $k<m$ is obtained by summing up $n_t(C_k,C_{m-k})$ or $n_t(C_{m-k},C_k)$ over all possible C_{m-k}'s. The product of conditional probabilities can be obtained with using the estimation $p_t(C_m)=n_t(C_m)/L$ for sufficiently large L, since the conditional probability $p_t(s_{i+m-1}|s_i,\ldots,s_{i+m-2})$ is given by

$$p_t(s_{i+m-1}|s_i,\ldots,s_{i+m-2})=\frac{p_t(s_i,\ldots,s_{i+m-2},s_{i+m-1})}{p_t(s_i,\ldots,s_{i+m-2})}. \quad (5)$$

We thus concentrate on the evolution of 3^m-dimensional vector $\boldsymbol{n}_t$ under Eqs. (2a) and (2b).

To investigate the time evolution of the system, we next consider the map associated with averaged $\boldsymbol{n}_t$ introduced in Ref. [9]. Denoting the operation of adding a particle to the cluster by T, the most probable value of $\boldsymbol{n}_t$ for a given $\boldsymbol{n}_0$ is defined by

$$\langle \boldsymbol{n}_t\rangle=\frac{1}{M_t}\sum T^t\boldsymbol{n}_0, \quad (6)$$

where M_t is the number of all possible growth paths starting from $\boldsymbol{n}_0$ and the sum is taken over all possible paths. Then $\langle \boldsymbol{n}_t\rangle$ gives rise to a deterministic time evolution for the stochastic growth starting with $\boldsymbol{n}_0$. Imagine that a pair of successive variables $(s,s')\in D$ in the set $\{s_1,s_2,\ldots,s_L\}$ becomes $(s+1,s'-1)$ at time t. Let $K_t(C_m;s,s')$ be the averaged value of $n_{t+1}(C_m)-n_t(C_m)$ when s and s' are given. Using $K_t(C_m;s,s')$, the most probable value of $n_{t+1}(C_m)$ for a given $n_t(C_m)$ can be expressed by

$$\langle n_{t+1}(C_m)\rangle=n_t(C_m)+\frac{\sum_{(s,s')\in D}K_t(C_m;s,s')n_t(s,s')}{\sum_{(s,s')\in D}n_t(s,s')}. \quad (7)$$

Here, $n_t(s,s')/\Sigma_{(s,s')\in D}n_t(s,s')$ is the probability of choosing (s,s') from the pairs in D. For periodic boundary conditions, one can set $s_m=s$ and $s_{m+1}=s'$ without loss of generality because of the translational invariance of the system. Then $K_t(C_m;s,s')$ is obtained from the change of the subsequence $C_{2m}=\{s_1,s_2,\ldots,s_{2m}\}$ between at time t and $t+1$. The difference of the number of $C_m=(\sigma_1,\sigma_2,\ldots,\sigma_m)$ between at t and $t+1$ is given by

$$\Delta(C_m;C_{2m})=\sum_{i=0}^{m}\left(\prod_{j=1}^{m}\delta_{\sigma_j,s_{i+j}+g_{i+j}}-\prod_{j=1}^{m}\delta_{\sigma_j,s_{i+j}}\right), \quad (8)$$

with $g_{i+j}=\delta_{i+j,m}-\delta_{i+j,m+1}$, and the conditional probability of finding C_{2m} for a given pair (s_m,s_{m+1}) is estimated by $n_t(C_{2m})/n_t(s_m,s_{m+1})$. These relations lead to the expression

$$K_t(C_m;s,s')=\sum_{C_{2m}}\delta_{s_m,s}\delta_{s_{m+1},s'}\sum_{i=0}^{m}\frac{\Delta(C_m;C_{2m})n_t(C_{2m})}{n_t(s_m,s_{m+1})}. \quad (9)$$

Performing the sum on C_{2m}, K_t is rewritten as

$$\begin{aligned}K_t(C_m;s,s')=\frac{1}{n_t(s,s')}\Bigg\{&(\delta_{\sigma_1,s'-1}-\delta_{\sigma_1,s'})n_t(s,s',\sigma_2,\ldots,\sigma_m)\\&+(\delta_{\sigma_m,s+1}-\delta_{\sigma_m,s})n_t(\sigma_1,\ldots,\sigma_{m-1},s,s')\\&+\sum_{i=1}^{m-1}(\delta_{\sigma_i,s+1}\delta_{\sigma_{i+1},s'-1}-\delta_{\sigma_i,s}\delta_{\sigma_{i+1},s'})\\&\times n_t(\sigma_1,\ldots,\sigma_{i-1},s,s',\sigma_{i+2},\ldots,\sigma_m)\Bigg\}. \quad (10)\end{aligned}$$

Using Eqs. (2a) and (2b), and the estimation of $p_t(C_m)=n_t(C_m)/L$, the numbers of subsequences with length $m+1$ in Eq. (10) are estimated by

$$n_t(s,s',\sigma_2,\ldots,\sigma_m)=\frac{n_t(s,s',\sigma_2,\ldots,\sigma_{m-1})n_t(s',\sigma_2,\ldots,\sigma_m)}{\sum_{\sigma_m}n_t(s',\sigma_2,\ldots,\sigma_m)}, \quad (11a)$$

$$n_t(\sigma_1,\ldots,\sigma_{m-1},s,s')=\frac{n_t(\sigma_1,\ldots,\sigma_{m-1},s)n_t(\sigma_2,\ldots,\sigma_{m-1},s,s')}{\sum_{\sigma_1}n_t(\sigma_1,\ldots,\sigma_{m-1},s)}. \quad (11b)$$

Substituting Eqs. (10), (11a), and (11b) into Eq. (7) gives $\langle n_t(C_m)\rangle$ as a function of $\boldsymbol{n}_t$. We here replace $\boldsymbol{n}_t$ with $\langle \boldsymbol{n}_t\rangle$, which simplifies the expression of the evolution equation although the validity of replacement should be checked numerically. Then we have a map $\langle \boldsymbol{n}_{t+1}\rangle=G(\langle \boldsymbol{n}_t\rangle)$ defined by

$$\begin{aligned}\langle n_{t+1}(C_m)\rangle=\langle n_t(C_m)\rangle+\frac{1}{\sum_{(s,s)\in D}\langle n_t(s,s')\rangle}\sum_{(s,s')\in D}\Bigg\{&(\delta_{\sigma_1,s'-1}-\delta_{\sigma_1,s'})\frac{\langle n_t(s,s',\sigma_2,\ldots,\sigma_{m-1})\rangle\langle n_t(s',\sigma_2,\ldots,\sigma_m)\rangle}{\sum_{\sigma_m}\langle n_t(s',\sigma_2,\ldots,\sigma_m)\rangle}\\&+(\delta_{\sigma_m,s+1}-\delta_{\sigma_m,s})\frac{\langle n_t(\sigma_1,\ldots,\sigma_{m-1},s)\rangle\langle n_t(\sigma_2,\ldots,\sigma_{m-1},s,s')\rangle}{\sum_{\sigma_1}\langle n_t(\sigma_1,\ldots,\sigma_{m-1},s)\rangle}\\&+\sum_{i=1}^{m-1}(\delta_{\sigma_i,s+1}\delta_{\sigma_{i+1},s'-1}-\delta_{\sigma_i,s}\delta_{\sigma_{i+1},s'})\langle n_t(\sigma_1,\ldots,\sigma_{i-1},s,s',\sigma_{i+2},\ldots,\sigma_m)\rangle\Bigg\}, \quad (12)\end{aligned}$$

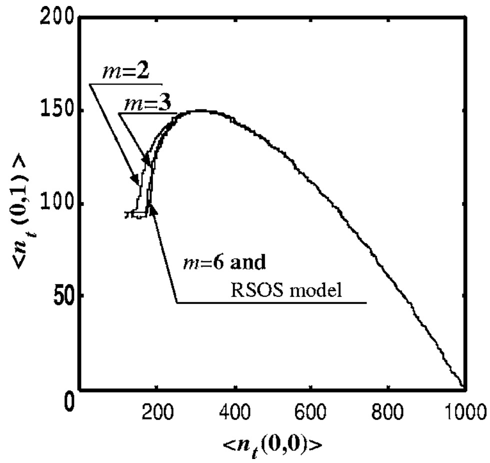

FIG. 2. Orbits of G projected on $[\langle n_t(0,0)\rangle, \langle n_t(0,1)\rangle]$ for $m=2,3,6$ and the PG path of the RSOS interface starting from a flat interface. It is hard to distinguish the growth path from the orbit of G for $m=6$ by the resolution of this figure.

with $C_m=(\sigma_1,\sigma_2,\ldots,\sigma_m)$. In actual iterations of G, if the denominator vanishes in each term of the right-hand side of Eq. (12), the corresponding term is set to zero. The solution $\langle \boldsymbol{n}^*\rangle$ of the fixed-point equation, or $\boldsymbol{p}^*=\langle \boldsymbol{n}^*\rangle/L$, gives us an estimation for the probability of finding a given surface in the steady state. Equations (2a) and (2b) can be easily extended in higher dimensions and the map can be obtained in a similar way.

In order to check the validity of the map G, we have carried out numerical simulations of the RSOS model with periodic boundary conditions and the map G from $m=2$ up to 6. Figure 2 shows the evolution of $[\langle n_t(0,0)\rangle, \langle n_t(0,1)\rangle]$ of the RSOS surface with an initially flat interface for $L=1000$ and the orbits of the map with the same initial condition, $n_0(0,0,\ldots,0)=1000$ and 0 otherwise. The orbits of G are obtained from 4×10^4 iterations of the map, and the PG path of the RSOS interface is estimated by the ensemble average over 8×10^3 samples. Our simulations show that the PG path of the RSOS surfaces is in excellent agreement with the corresponding orbit of the map for $m=6$. We have also checked the fixed point $\boldsymbol{n}^*$ of the map with the probability p^* of finding a subsequence in the steady state. The results for $L=1000$ are summarized in Table I. p^* is estimated numerically by the values after 10^5 iterations of the map. The fixed point is close to the phase point corresponding to the uncorrelated random state, $p^*(C_m)=1/3^m$ for all C_m's, and the obtained probabilities $p^*(0,0)$ and $p^*(0,1)$ for $m=6$ agree with those of the RSOS model within 0.2%.

We next demonstrate that the map G for $m=6$ well describes the averaged growth path for any given initial condition. Figure 3(a) shows the evolution of the vector $[\langle n_t(0,0)\rangle, \langle n_t(-1,0)\rangle]$ in the RSOS model for $L=100$ and Fig. 3(b) shows the corresponding orbits of the map for $m=6$. The initial conditions are chosen randomly from RSOS clusters consisting of 10^5 particles and each orbit is averaged over 8×10^3 samples in Fig. 3(a). The phase portrait of the map is in good agreement with that of the RSOS model.

TABLE I. Estimation of probabilities in the steady state for $L=1000$. The probability $p^*(C_2)$ is estimated by $n^*(C_2)/L$ for $C_2=\{0,0\}$ and $\{0,1\}$. Numerical results for the RSOS model are also shown for comparison.

m	$p^*(0,0)$	$p^*(0,1)$
2	0.119870	0.093592
3	0.116475	0.094384
4	0.115761	0.094392
5	0.115518	0.094372
6	0.115403	0.094362
RSOS model	0.115295	0.094367

III. DISCUSSION AND SUMMARY

We consider the statistical property of height fluctuations associated with the fixed point of the map G. Since the height at site $i+\ell$ in a $(1+1)$-dimensional system is given by $h_{i+\ell}=h_i+s_{i+1}+s_{i+2}+\cdots+s_{i+\ell}$, the height-height correlation function $C_h(\ell)$ is given by

$$C_h(\ell)^2 \equiv \langle\!\langle (h_{i+\ell}-h_i)^2 \rangle\!\rangle = \ell \langle\!\langle s_i^2 \rangle\!\rangle + 2\sum_{j=1}^{\ell-1} (\ell-j) \langle\!\langle s_i s_{i+j} \rangle\!\rangle , \qquad (13)$$

which shows that the height-height correlation function is

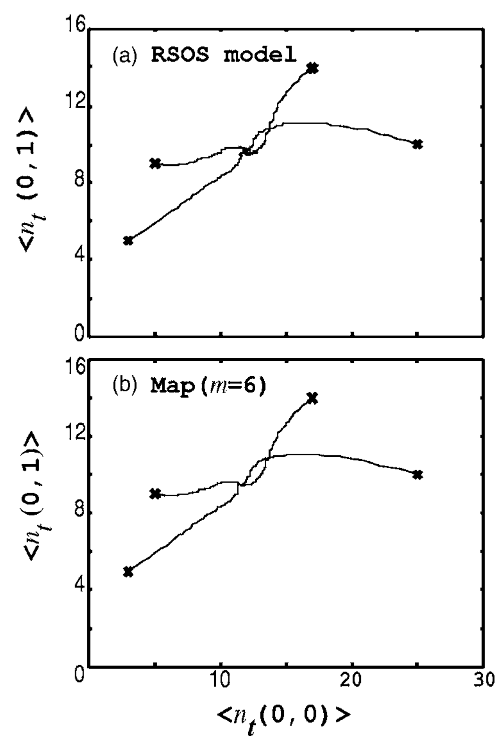

FIG. 3. Time evolution of $[\langle n_t(0,0)\rangle, \langle n_t(1,0)\rangle]$ of the RSOS model and the map G for $L=100$. The symbol $\times$ denotes the initial phase point for each orbit. (a) Orbits of the RSOS model. (b) Orbits of the map G for $m=6$ with the same initial conditions in (a).

determined by $C_s(j)$. The roughness exponent α can be estimated from the behavior of $C_h(\ell) \sim \ell^{\alpha}$ and Eq. (13) can be easily extended in higher dimensions. For the short-range interaction, the average $\langle\!\langle s_i s_{i+j} \rangle\!\rangle$ in Eq. (13) can be obtained from the second largest eigenvalue of a transfer matrix V defined by

$$V_{C'C} = \left(\prod_{j=1}^{m-2} \delta_{u'_j, u_{j+1}} \right) p^*(u'_{m-1}|C), \tag{14}$$

with $C=\{u_1, \ldots, u_{m-1}\}$ and $C'=\{u'_1, \ldots, u'_{m-1}\}$. Using the transfer matrix, the fixed point $\boldsymbol{p}^*$ gives us an estimation for averaged physical quantities under consideration in the steady state. Let λ_i be the ith eigenvalue of V with $\lambda_1 \geqslant \lambda_2 \geqslant \cdots$. From the relations $p^*(C') = \Sigma_C V(0)_{C'C} p^*(C)$ and $\Sigma_{C'} V(0)_{C'C} = 1$, it is easy to show that the largest eigenvalue λ_1 is unity and $1 \geqslant |\lambda_i|$ holds for any i. When the invariant probability measure of the system is unique, λ_1 is greater than λ_2 for finite m. Then $\langle\!\langle s_i s_{i+j} \rangle\!\rangle$ is estimated by λ_2^{j-1} for sufficiently large L, which leads to the estimation that $C_h(\ell)^2$ is of order of ℓ from Eq. (13). It follows that, for large ℓ and L

$$C_h(\ell) \propto \ell^{1/2}, \tag{15}$$

so we have $\alpha=1/2$ for interfaces described by a Markov process in 1+1 dimensions. Note that the value of $\alpha=1/2$ is also obtained from interfaces generated by uncorrelated jumps of height, which is the limiting case of $p(\Delta h_2, \Delta x_2 | \Delta h_1, \Delta x_1) = p(\Delta h_2, \Delta x_2) p(\Delta h_1, \Delta x_1)$ in Eq. (1). From an extended identity of Eq. (13) in higher dimensions, one can see that the $C_h(r)^2$ is bounded by r if $C_s(r) < 0$ except $r=0$, where r is the distance between sites. In such systems the roughness exponent lies in the interval $0 \leqslant \alpha \leqslant 1/2$. We have investigated $C_s(r)$ numerically in a (2+1)-dimensional RSOS model and the result shows that $C_s(r)$ is negative for $r > 0$. It is of interest to investigate the Markov property in spatial dimensions in the growth process showing $\alpha=1/2$ in 1+1 dimensions and $\alpha < 1/2$ in 2+1 dimensions [10,13,14,18–22]. It should be noted that the estimation of α in 2+1 dimensions is not as easy as in 1+1 dimensions even for the system with short-range interaction. V is defined as a row-to-row transfer matrix and the size of the matrix becomes infinitely large in the limit L goes to infinity. The relation between the value of α in 2+1 dimensions and the Markov property of conditional probabilities is an open problem at the present stage.

In an analogy of statistical mechanical physics, our method is equivalent with introducing a Hamiltonian to the system in 1+1 dimensions, defined by

$$p_t(s_1, s_2, \ldots, s_L) = Z^{-1} \exp[-H(s_1, \ldots, s_L)], \tag{16}$$

with $Z = \Sigma \exp(-H)$. In the case of short-range interaction, H is given as a sum of H_i defined by

$$H_i(s_i, s_{i+1}, \ldots, s_{i+m-1}) = -\ln p_t(s_{i+m-1} | s_i, s_{i+1}, \ldots, s_{i+m-2}). \tag{17}$$

For the RSOS model the Hamiltonian is expanded in the form

$$H = J_1(t) \sum_i s_i + J_2(t) \sum_i s_i^2 + J_3(t) \sum_i s_i s_{i+1} + J_4(t) \sum_i s_i s_{i+1}^2 + J_5(t) \sum_i s_i^2 s_{i+1} + \cdots. \tag{18}$$

Here, $\boldsymbol{J}(t) = [J_1(t), J_2(t), \ldots]$ can be determined from $\langle \boldsymbol{n}_t \rangle$ and $\boldsymbol{J} = (0, 0, \ldots)$ corresponds to the uncorrelated state. From the form of H it is almost trivial that $\alpha=1/2$ for systems with the short-range interaction. There exist other growth models showing behaviors characterized by $\alpha \neq 1/2$ in 1+1 dimensions [1]. For these growth models we can conclude that the interaction between local slopes is not of the short-range type in observed length scales, that is, the interaction length is $m \sim L$ or infinite. In the case of long crossovers the situation is more complicated than that of the growth discussed here and the scaling property may be described by a generalized scaling function with local scaling indices [15–17].

In 2+1 dimensions, the estimation of α is a formidable task for the system under consideration, as has been mentioned before. However, we can obtain some insight on the roughness exponent if the system has a Markov property for conditional probabilities. Numerical simulations have shown that $\alpha < 1/2$ in several growth models, and $\alpha \neq 1/2$ suggests that the correlation function of height differences decays algebraically. In our method, that algebraic behavior is understandable if the system reaches a critical state. The property of becoming critical without adjusting external parameters has been referred to as self-organized criticality [23]. This criticality of the surface roughening can be discussed more concretely for the single-step model [18]. In the single-step model a surface grows and roughenss under the restriction that the height of the deposit differs from the height of its nearest neighbors by exactly one lattice unit, and in 2+1 dimensions the single-step interface can be transformed to the six-vertex model by mapping height differences to incoming and outgoing arrows. In the six-vertex model there are four phases in terms of vertex energies, or Boltzmann weights [24]. We expect that the fixed point of the map G for the (2+1)-dimensional single-step growth corresponds to a phase point on a critical line between phases in the parameter space of vertex energies, which is planned to report elsewhere.

In summary, we have presented a method of extracting an approximated evolution equation for the distribution of height fluctuations from growth rules in stochastic discrete models generating rough surfaces with a Markov property in spatial dimensions. Using the Markov property of the surface, it has been shown that the evolution equation of fluctuating field can be described by a map with finite degrees of freedom. Applying the method to a (1+1)-dimensional RSOS model, the map has been derived analytically under assumptions on conditional probabilities. The validity of the obtained map has been checked numerically and the results are in good agreement with the averaged evolution of RSOS interfaces. In an analogy of statistical mechanics, our method is equivalent to introducing a Hamiltonian with finite terms to the system under consideration. From a naive consideration on the correlation function, our method suggests the self-organized criticality in 2+1 dimensions. The method presented in this work will provide us with a useful tool to investigate the probability distribution of height fluctuations.

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